INTERNATIONAL ENERGY AGENCY
energy conservation in buildings and
community systems programme

Technical Note AIVC 28

A guide to air change efficiency

February 1990

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This report is part of the work of the IEA Energy Conservation in Buildings & Community Systems Programme.

Annex V Air Infiltration and Ventilation Centre

Document AIC-TN-28-90
ISBN 0 946075 43 3

Participants in this task:

Belgium, Canada, Denmark, Federal Republic of Germany, Finland, Italy, Netherlands, New Zealand, Norway, Sweden, Switzerland, United Kingdom, and United States of America

Distribution: Annex participants only

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A guide to air change efficiency

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PREFACE

International Energy Agency

The International Energy Agency (IEA) was established in 1974 within the framework of the Organisation for Economic Co-operation and Development (OECD) to implement an International Energy Programme. A basic aim of the IEA is to foster co-operation among the twenty-one IEA Participating Countries to increase energy security through energy conservation, development of alternative energy sources and energy research development and demonstration (RD&D). This is achieved in part through a programme of collaborative RD&D consisting of forty-two Implementing Agreements, containing a total of over eighty separate energy RD&D projects. This publication forms one element of this programme.

Energy Conservation in Buildings and Community Systems

The IEA sponsors research and development in a number of areas related to energy. In one of these areas, energy conservation in buildings, the IEA is sponsoring various exercises to predict more accurately the energy use of buildings, including comparison of existing computer programs, building monitoring, comparison of calculation methods, as well as air quality and studies of occupancy. Seventeen countries have elected to participate in this area and have designated contracting parties to the Implementing Agreement covering collaborative research in this area. The designation by governments of a number of private organisations, as well as universities and government laboratories, as contracting parties, has provided a broader range of expertise to tackle the projects in the different technology areas than would have been the case if participation was restricted to governments. The importance of associating industry with government sponsored energy research and development is recognised in the IEA, and every effort is made to encourage this trend.

The Executive Committee

Overall control of the programme is maintained by an Executive Committee, which not only monitors existing projects but identifies new areas where collaborative effort may be beneficial. The Executive Committee ensures that all projects fit into a pre-determined strategy, without unnecessary overlap or duplication but with effective liaison and communication. The Executive Committee has initiated the following projects to date (completed projects are identified by *):

I. Load Energy Determination of Buildings *

II. Ekistics and Advanced Community Energy Systems *

III. Energy Conservation in Residential Buildings *

IV. Glasgow Commercial Building Monitoring *

V. Air Infiltration and Ventilation Centre

VI. Energy Systems and Design of Communities *
VII. Local Government Energy Planning *
VIII. Inhabitant Behaviour with Regard to Ventilation *
IX. Minimum Ventilation Rates *
X. Building HVAC Systems Simulation
XI. Energy Auditing *
XII. Windows and Fenestration *
XIII. Energy Management in Hospitals *
XIV. Condensation
XV. Energy Efficiency in Schools
XVI. BEMS – 1: Energy Management Procedures
XVII. BEMS – 2: Evaluation and Emulation Techniques
XVIII. Demand Controlled Ventilating Systems
XIX. Low Slope Roof Systems
XX. Air Flow Patterns within Buildings
XXI. Energy Efficient Communities
XXII. Thermal Modelling

Annex V – Air Infiltration and Ventilation Centre

The IEA Executive Committee (Building and Community Systems) has highlighted areas where the level of knowledge is unsatisfactory and there was unanimous agreement that infiltration was the area about which least was known. An infiltration group was formed drawing experts from most progressive countries, their long term aim to encourage joint international research and increase the world pool of knowledge on infiltration and ventilation. Much valuable but sporadic and unco-ordinated research was already taking place and after some initial groundwork the experts group recommended to their executive the formation of an Air Infiltration and Ventilation Centre. This recommendation was accepted and proposals for its establishment were invited internationally.

The aims of the Centre are the standardisation of techniques, the validation of models, the catalogue and transfer of information, and the encouragement of research. It is intended to be a review body for current world research, to ensure full dissemination of this research and based on a knowledge of work already done to give direction and firm basis for future research in the Participating Countries.

The Participants in this task are:

- Belgium
- Canada
- Denmark
- Federal Republic of Germany
- Finland
- Italy
- Netherlands
- New Zealand
- Norway
- Sweden
- Switzerland
- United Kingdom
- United States of America

NOMENCLATURE

Symbols		**Units**
t	time	s
$\bar{\tau}_p$	local mean age of air at point p	s
$\bar{\tau}_e$	local mean age of air at the exhaust duct	s
$C_p(t)$	concentration of tracer at point p at time t	
$C_e(t)$	concentration of tracer at exhaust at time t	
C_s	concentration of tracer in supply duct	
$C(o)$	initial concentration of tracer in room	
$<C(t)>$	room mean concentration of tracer	
μ^n	nth moment	
$<\bar{\tau}>$	room mean age of air	s
V	room volume	m^3
Q	airflow rate from supply duct	m^3/s
n	specific flow	s^{-1}
τ_n	nominal time constant	s
$\bar{\tau}_r$	air change time	s
ε_a	air change efficiency	%
η	coefficient of air change performance	
ε_p	local air change index	

1. INTRODUCTION

The main objective of this report was to provide a concise introduction into the subject of air change efficiency. Existing literature in this subject area is extensive, but it tends to be very detailed and is difficult for a newcomer to understand. Different authors also use different symbols and/or different definitions for the same concepts, which tends to confuse the reader. Little has been produced covering the basic ideas and concepts behind some of the terms used. Therefore this report aims to show the origins of the concepts used, provide proofs of the basic formulae and suggests standard symbols and definitions.

Sandberg and Skåret [1] differentiate between the terms air change efficiency and ventilation efficiency. Air change efficiency is a measure of how effectively the air present in a room is replaced by fresh air from the ventilation system whereas ventilation efficiency is a measure of how quickly a contaminant is removed from the room. This report covers only air change efficiency and related concepts. It should also be noted that the theory described in this report assumes a mechanically ventilated, air tight room where all the air enters and leaves via designated inlet and exhaust ducts.

The list of references given at the back of this report is not intended to be exhaustive in the subject area of air change efficiency, it is merely a selected list. A complete bibliography can be obtained via AIRBASE.

2. FREQUENCY DISTRIBUTION

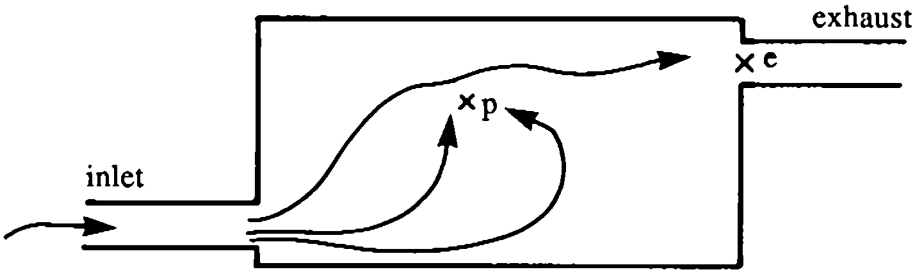

Fig. 1 Representation of a mechanically ventilated room

Figure 1 represents a mechanically ventilated room with one inlet duct and one exhaust duct. Consider a small volume, δV, at an arbitrary point p in the room. Consider also a small packet of air entering the room at the inlet duct, at time t = 0. Some of the molecules of air in this packet will pass through the point p. Because the molecules will travel by different routes, the number arriving at p will vary with time, typically in the manner shown in figure 2. This is the frequency distribution of the molecules arriving at p from a single packet of air entering the room at time zero.

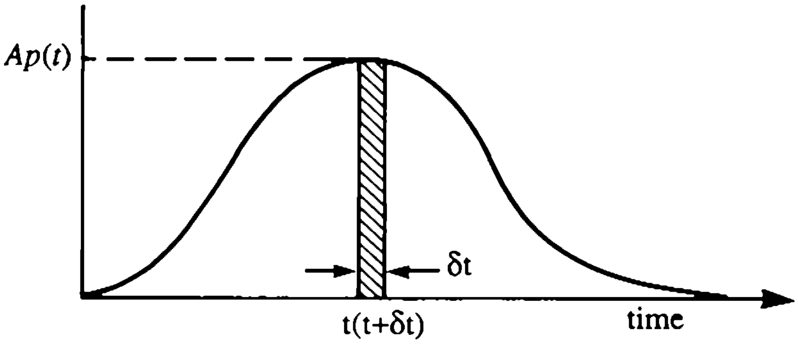

Fig. 2 The frequency distribution curve for air arriving at a point

The number of molecules arriving at point p, between time t and (t + δt) is

Area of the shaded strip = $A_p(t).\delta t$

The total number of molecules arriving between time 0 and t is

$$F_p(t) = \int_0^t A_p(t)\,dt \qquad (1)$$

The total number of molecules which will arrive at p is

$$\int_0^\infty A_p(t)\,dt = \text{Area under the frequency distribution curve}$$

If the frequency distribution is expressed in terms of the fraction of the total number of molecules arriving at p, then the total number of molecules to arrive at p is 1, or 100% and so

$$\text{Area under the curve} = \int_0^\infty A_p(t)\,dt = 1 \tag{2}$$

3. LOCAL MEAN AGE OF AIR ($\bar{\tau}_p$)

The local mean age of air is defined as the average time it takes for air to travel from the inlet to any point p in the room. The mean age of the air at the point p can be found from the centroid of the frequency distribution curve, by taking moments about the vertical axis.

$$\text{The mean age at p,}\quad \bar{\tau}_p = \frac{\int_0^\infty t.A_p(t)\,dt}{\int_0^\infty A_p(t)\,dt} \tag{3}$$

From equation 2,

$$\bar{\tau}_p = \int_0^\infty t.A_p(t)\,dt \tag{4}$$

The local mean age of air will be different for different points in the room. For example, the mean age of the air at the exhaust duct can be expressed by

$$\bar{\tau}_e = \int_0^\infty t.A_e(t)\,dt \tag{5}$$

4. MEASUREMENT OF LOCAL MEAN AGE

The local mean age of air within the room can be measured using three different tracer gas techniques. These are the pulse method, the tracer step-up method and the tracer decay method.

4.1 Pulse Method

A short pulse of tracer gas is injected into the air in the inlet duct as it enters the room. It is assumed that the air and tracer are fully mixed before entering the room and that they migrate together once in the room. If the injection time is short enough, then only the packet of air which enters at time zero contains tracer gas. Hence the tracer gas concentration at point p will imitate exactly the frequency distribution curve such that equation 3 becomes

$$\bar{\tau}_p = \frac{\int_0^\infty t.C_p(t)\,dt}{\int_0^\infty C_p(t)\,dt} \qquad (6)$$

where $C_p(t)$ is the concentration of tracer gas at point p at time t.

4.2 Tracer Step-up Method

Tracer gas is injected at a constant rate into the inlet duct. It is assumed that the tracer and air are fully mixed before entering the room, to produce a steady concentration of C_s at the inlet.

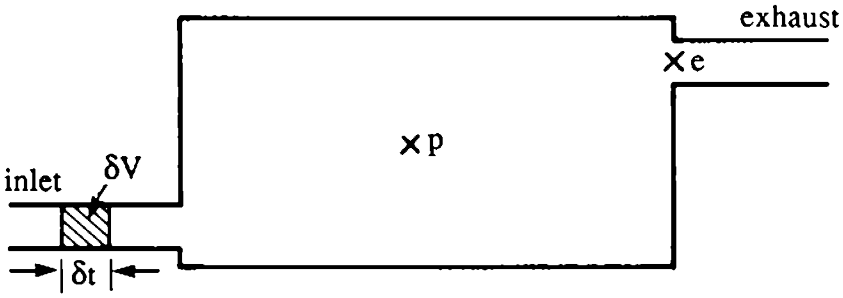

Fig. 3 The tracer step-up method

Consider a small packet of air, of volume δV, in the inlet duct. This packet contains tracer gas of concentration C_s, and will enter the room at time t = 0, with a duration of δt.

At time t, the proportion of air from this packet which has reached the point p in the time interval t to (t + δt) is,

$$A_p(t)\ \delta t$$

The concentration of tracer in this air is $C_s A_p(t)\ \delta t$

Therefore the concentration of tracer at the point p, due only to this packet is

$$\delta C_p = C_s A_p(t)\ \delta t$$

The continuous injection of tracer into the supply duct can be considered as small packets entering the room sequentially so that the concentration of tracer at the point p is due to all packets that have entered up to this time, t.

Therefore

$$C_p(t) = \int_0^t \delta C_p \, dt = \int_0^t C_s A_p(t) \, \delta t$$

Hence

$$\frac{C_p(t)}{C_s} = \int_0^t A_p(t) \, dt \tag{7}$$

Differentiating gives

$$A_p(t) = \frac{C_p'(t)}{C_s} \tag{8}$$

By substituting for $A_p(t)$ in equation 4, the local mean age at point p may be written

$$\bar{\tau}_p = \int_0^\infty t \, \frac{C_p'(t)}{C_s} \, dt \tag{9}$$

Then from Appendix A, equation 9 becomes

$$\bar{\tau}_p = \int_0^\infty \left(1 - \frac{C_p(t)}{C_s} \right) dt \tag{10}$$

since, in this case K = 0 and $f(x) = \dfrac{C_p(t)}{C_s}$. Also since the concentration at the exhaust duct at time infinity, $C_e(\infty)$ is equal to C_s, then $f(\infty) = 1$, and so A = 1.

This holds for any point in the room, therefore the local mean age of air at the exhaust duct (i.e. point e) is given by

$$\bar{\tau}_e = \int_0^\infty \left(1 - \frac{C_e(t)}{C_s} \right) dt \tag{11}$$

4.3 Tracer Decay Method

The room contains a uniform concentration of tracer gas $C(o)$, at time t = 0. The tracer is then allowed to decay, by replacement with air from the supply duct, so that at any time t, the material in any volume δV will be a mixture of tracer gas and air.

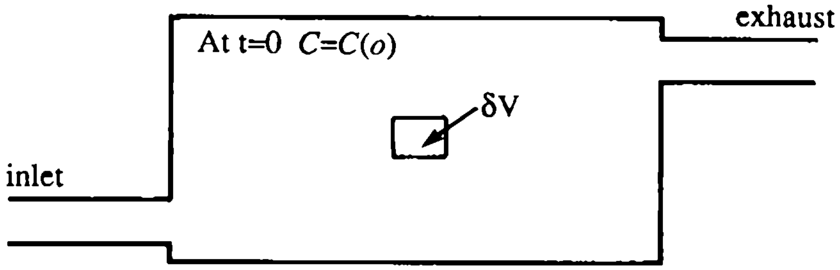

Fig. 4 Tracer Decay Method

Consider a small volume δV at some point in the room. At time t, the total amount of air within δV due to all packets of air that have entered the room since time t = 0, is

$$\int_0^t A_p(t)\,dt = F_p(t) \tag{1}$$

The remainder of the material present in the volume δV must have originated in the room and must therefore contain tracer gas at the initial concentration, $C(o)$. Therefore the quantity of tracer in δV is

$$C_p(t) = C(o)\left(1 - F_p(t)\right) \tag{12}$$

Therefore

$$F_p(t) = 1 - \frac{C_p(t)}{C(o)}$$

From equation 1, this becomes

$$\int_0^t A_p(t)\,dt = 1 - \frac{C_p(t)}{C(o)} \tag{13}$$

Differentiating equation 13, gives

$$A_p(t) = -\frac{C_p'(t)}{C(o)} \tag{14}$$

Substituting for $A_p(t)$ in equation 4, gives

$$\bar{\tau}_p = -\int_0^{\infty} t.\frac{C_p'(t)}{C(o)}\,dt \qquad (15)$$

Then from Appendix A, equation 15 becomes

$$\bar{\tau}_p = \int_0^{\infty} \frac{C_p(t)}{C(o)}\,dt \qquad (16)$$

since, in this case K= 0, and $f(x) = \frac{C_p(t)}{C(o)}$. Also since the concentration at the exhaust duct at time infinity, $C_e(\infty)$ is equal to zero, then $f(\infty) = 0$ and so A = 0.

Equation 16 will hold true for any small volume (point) within the room, and can therefore be applied to the exhaust duct such that equation 16 becomes

$$\bar{\tau}_e = \int_0^{\infty} \frac{C_e(t)}{C(o)}\,dt \qquad (17)$$

5. THE USE OF MOMENTS

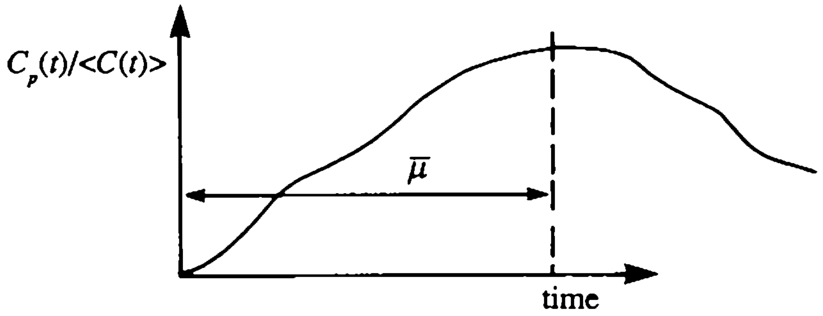

Fig. 5 The determination of moments

From the graph of measured concentration versus time at a point in the room, the nth moment about the y-axis is given by

$$\mu_c^n = \int_0^{\infty} t^n\, C_p(t)\,dt \qquad (18)$$

The room average concentration $<C(t)>$ at time t is by definition

$$< C(t) > = \frac{1}{V}\int C_p(t)\,dV \qquad (19)$$

The n^{th} moment about the y-axis of the room average concentration distribution graph is given by

$$\mu^{n}_{<c>} = \int_{0}^{\infty} t^{n} < C(t) > dt \tag{20}$$

Substituting equation 19 into equation 20 gives

$$\mu^{n}_{<c>} = \int_{0}^{\infty} t^{n} \left[\frac{1}{V} \int C_{p}(t)\, dV \right] dt$$

Rearranging the order of integration

$$\mu^{n}_{<c>} = \frac{1}{V} \int \left[\int_{0}^{\infty} t^{n}\, C_{p}(t)\, dt \right] dV$$

Substituting equation 18 gives

$$\mu^{n}_{<c>} = \frac{1}{V} \int \mu^{n}_{c}\, dV = < \mu^{n}_{c} > \tag{21}$$

The quantity $\mu^{n}_{<c>}$ is the average of the room moments at all points in the room. Therefore equation 21 shows that the moments of the room average equal the average of the room moments.

6. THE ROOM MEAN AGE ($< \bar{\tau} >$)

The room mean age of air is defined as the average value of the local mean ages of air for all points in a room.

Since the moments of the room average equals the average of the room moments we can express the room mean age of air in terms of the room average concentration of tracer gas, for each of the three methods.

(i) **Pulse method**

from equation 6, the local mean age of air at point p is given by

$$\bar{\tau}_{p} = \frac{\int_{0}^{\infty} t . C_{p}(t)\, dt}{\int_{0}^{\infty} C_{p}(t)\, dt}$$

Therefore the room mean age can be expressed by

$$< \bar{\tau} > = \frac{\int_0^\infty t.< C(t) > dt}{\int_0^\infty < C_p(t) > dt} \tag{22}$$

Unfortunately $<C_p(t)>$ cannot be easily measured. However since all the material in the room must eventually enter the exhaust duct

$$\int_0^\infty < C_p(t) > dt = \int_0^\infty C_e(t)\, dt$$

Therefore equation 22 becomes

$$< \bar{\tau} > = \frac{\int_0^\infty t.< C(t) > dt}{\int_0^\infty C_e(t)\, dt} \tag{23}$$

(ii) **Tracer Step-Up Method**

from equation 10, the local mean age of air is given by

$$\bar{\tau}_p = \int_0^\infty \left(1 - \frac{C_p(t)}{C_s} \right) dt$$

Therefore the room mean age can be expressed by

$$< \bar{\tau} > = \int_0^\infty \left(1 - \frac{< C(t) >}{C_s} \right) dt \tag{24}$$

(iii) **Tracer Decay Method**

from equation 16, the local mean age of air is given by

$$\bar{\tau}_p = \int_0^\infty \frac{C_p(t)}{C(o)}\, dt$$

Therefore

$$< \bar{\tau} > = \int_0^\infty \frac{< C(t) >}{C(o)}\, dt \tag{25}$$

Because the room average concentration cannot easily be measured, it is necessary to express the room mean ages in terms of a measurable quantity, such as the tracer gas concentration in the exhaust duct. This requires a mass balance equation to be constructed for the room.

6.1 Mass Balance Equation

The mass balance for the room, at time t, can be expressed by equating the rate of volumetric change of tracer concentration within the room to the rate of tracer entering the room minus the rate leaving.

Thus

$$V\frac{d}{dt}<C(t)>=Q(C_s-C_e(t))$$

where V is the volume of the room (m^3)
Q is the airflow rate into the room (m^3/s)

Multiplying equation 26 by $t^{n+1}$and integrating gives

$$\int_0^\infty t^{n+1}\frac{d}{dt}<C(t)>dt=\frac{Q}{V}\int_0^\infty t^{n+1}(C_s-C_e(t))\,dt \qquad (27)$$

from Appendix A, equation 27 becomes

$$(n+1)\int_0^\infty t^n(C(\infty)-<C(t)>)\,dt=\frac{Q}{V}\int_0^\infty t^{n+1}(C_s-C_e(t))\,dt \qquad (28)$$

Then, by converting the left hand side of this equation into each of the room mean age expressions found earlier, we can obtain equations for the room mean age in terms of the exhaust concentrations of tracer gas for each of the measurement methods.

6.2 Pulse Method

For the pulse method, $C_s=0$, $C(\infty)=0$ and using n = 1, equation 28 becomes

$$\int_0^\infty t<C(t)>dt=\frac{Q}{2V}\int_0^\infty t^2C_e(t)\,dt \qquad (29)$$

Therefore equation 23 becomes

$$< \bar{\tau} > = \frac{Q}{2V} \frac{\int_0^\infty t^2 C_e(t)\,dt}{\int_0^\infty C_e(t)\,dt} \tag{30}$$

6.3 Tracer Step-Up Method

For the tracer step-up method, $C(\infty) = C_s$ and using n = 0 equation 28 becomes

$$\int_0^\infty (C_s - < C(t) >)\,dt = \frac{Q}{V} \int_0^\infty t(C_s - C_e(t))\,dt \tag{31}$$

Therefore

$$\int_0^\infty \left(1 - \frac{< C(t) >}{C_s}\right) dt = \frac{Q}{V} \int_0^\infty t \left(1 - \frac{C_e(t)}{C_s}\right) dt$$

Therefore equation 24 becomes

$$< \bar{\tau} > = \frac{Q}{V} \int_0^\infty t \left(1 - \frac{C_e(t)}{C_s}\right) dt \tag{32}$$

6.4 Tracer Decay Method

For the tracer decay method, $C_s = 0$, $C(\infty) = 0$ and using n = 0 equation 28 becomes

$$\int_0^\infty < C(t) > dt = \frac{Q}{V} \int_0^\infty t.C_e(t)\,dt \tag{33}$$

Therefore equation 25 becomes

$$< \bar{\tau} > = \frac{Q}{V} \int_0^\infty t \frac{C_e(t)}{C(o)}\,dt \tag{34}$$

An alternative form of equation 34 may be obtained by constructing a volumetric balance of tracer gas at the exhaust at time t.

The quantity of tracer leaving in time $\delta t = QC_e(t)\,\delta t$

Therefore the total quantity leaving the room $= \int_0^\infty QC_e(t)\,dt$

The total quantity of tracer which will leave the room, in infinite time, must be equal to the quantity of tracer gas initially present since this is a tracer decay test. The initial quantity of tracer is *C(o)V*, and therefore

$$C(o)V = \int_0^\infty QC_e(t)\,dt$$

Substituting this result into equation 34 gives

$$<\bar{\tau}> = \frac{\int_0^\infty t.C_e(t)\,dt}{\int_0^\infty C_e(t)\,dt} \qquad (35)$$

SUMMARY OF MEAN AGE EQUATIONS

LOCAL MEAN AGE $\bar{\tau}$

1. pulse method

$$\bar{\tau}_p = \frac{\int_0^\infty t.C_p(t)\,dt}{\int_0^\infty C_p(t)\,dt}$$

2. step-up method

$$\bar{\tau}_p = \int_0^\infty \left(1 - \frac{C_p(t)}{C_s}\right) dt$$

3. decay method

$$\bar{\tau}_p = \int_0^\infty \frac{C_p(t)}{C(o)}\,dt$$

ROOM MEAN AGE $< \bar{\tau} >$

1. pulse method

$$< \bar{\tau} > = \frac{Q}{2V} \frac{\int_0^\infty t^2 C_e(t)\,dt}{\int_0^\infty C_e(t)\,dt}$$

2. step-up method

$$< \bar{\tau} > = \frac{Q}{V} \int_0^\infty t \left(1 - \frac{C_e(t)}{C_s}\right) dt$$

3. decay method

$$< \bar{\tau} > = \frac{Q}{V} \int_0^\infty t \frac{C_e(t)}{C(o)}\,dt$$

$$< \bar{\tau} > = \frac{\int_0^\infty t.C_e(t)\,dt}{\int_0^\infty C_e(t)\,dt}$$

7. AIR INDICES

7.1 Specific Flow (n)

The specific flow is defined as the total volumetric supply airflow per unit volume of the room

$$n = \frac{Q}{V} \tag{36}$$

where Q = ventilation airflow (m^3/s)
V = total room volume (m^3)

Specific flow is often called the air change rate.

7.2 Nominal Time Constant (τ_n)

This is the inverse of the specific flow

$$\tau_n = \frac{V}{Q} \tag{37}$$

Under piston conditions, τ_n is the time it will take to exchange all the air in the room for fresh air.

7.3 Air Change time ($\bar{\tau}_r$)

The air change time for all the air in the room is equal to twice the room mean age of all the air in the room

$$\bar{\tau}_r = 2 < \bar{\tau} > \tag{38}$$

where $< \bar{\tau} >$ = room mean age

The proof of this relationship, adapted from Sandberg [4] is given in Appendix B.

7.4 Air Change Efficiency (ε_a)

The air change efficiency is defined as the ratio between the nominal time constant, τ_n, and the air change time, $\bar{\tau}_r$, for the room, and is a measure of how quickly the air in the room is replaced.

$$\varepsilon_a = \frac{\tau_n}{\bar{\tau}_r} \times 100 \qquad (39)$$

where τ_n = nominal time constant
$\bar{\tau}_r$ = air change time

7.5 Coefficient of Air Change Performance (η)

The coefficient of air change performance is defined as the ratio between the nominal time constant and the room mean age

$$\eta = \frac{\tau_n}{<\bar{\tau}>}$$

where τ_n = nominal time constant
$<\bar{\tau}>$ = room mean age

It should be noted that the coefficient of air change performance equals twice the air change efficiency

$$\eta = 2\varepsilon_a$$

This term is equivalent to the ventilation effectiveness definition of ASHRAE Standard 62-1989 "Ventilation for Acceptable Indoor Air Quality".

7.6 Local Air Change Index (ε_p)

This index characterises the conditions at a particular point and may be large due to the position in the room of the measurement point.

$$\varepsilon_p = \frac{\tau_n}{\bar{\tau}_p} \qquad (40)$$

where τ_n = nominal time constant
$\bar{\tau}_p$ = local mean age of air at point p.

8. DEMONSTRATION OF THE EVALUATION OF AIR INDICES FOR TWO TYPICAL CASES

The local mean ages of air at different points within the room, the room mean age of the air and air change efficiencies can be determined for two ideal cases, (a) piston flow and (b) complete mixing. For both cases it is assumed that tracer decay tests are performed where initially the room contains an homogeneous concentration of tracer gas. The tracer concentration is then allowed to decay as the air in the room is replaced by the ventilation airflow.

8.1 Piston Flow

This may also be known as 'plug flow' or 'displacement flow' and is regarded as the most efficient form of ventilation. The ventilation airflow acts as a piston, which pushes the 'old' air in the room in front of it without actually mixing. Therefore, all the air which reaches an arbitrary point from a small packet of fresh air at the inlet does so at the same time. This time is by definition, the local mean age of air at that point, τ_p. This is illustrated in figure 6

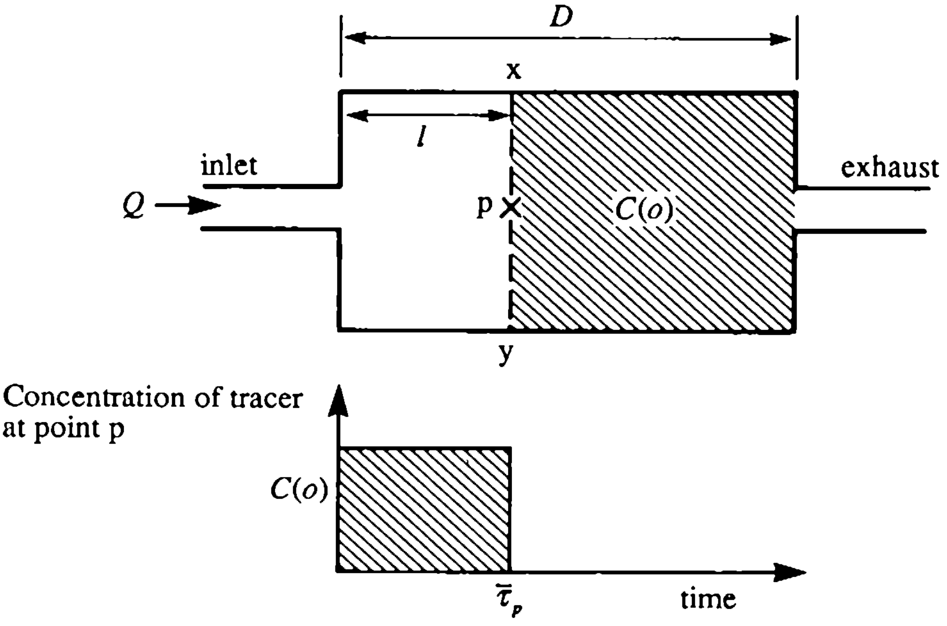

Fig. 6 Piston flow

For ideal piston flow in a room of uniform cross section A, the local mean ages of air at all the points on the line x→y (i.e. in a line with the point p, figure 6) will have the same local mean age as at p (i.e. $\bar{\tau}_p$)

The velocity, u, of the air along the room is

$$u = \frac{Q}{A}$$

Therefore, the mean age at p, which is the time taken for air from the inlet duct to travel the distance *l* to the point p, is

$$\bar{\tau}_p = \frac{l}{u} = \frac{lA}{Q}$$

The nominal time constant, $\tau_n = \dfrac{V}{Q}$, and the room volume V = DA. Hence

$$\bar{\tau}_p = \frac{lA}{Q} = \frac{lA\tau_n}{V} = \frac{lA\tau_n}{DA} = \frac{l\tau_n}{D}$$

Thus the mean age varies linearly with distance from the inlet duct, as shown in figure 7.

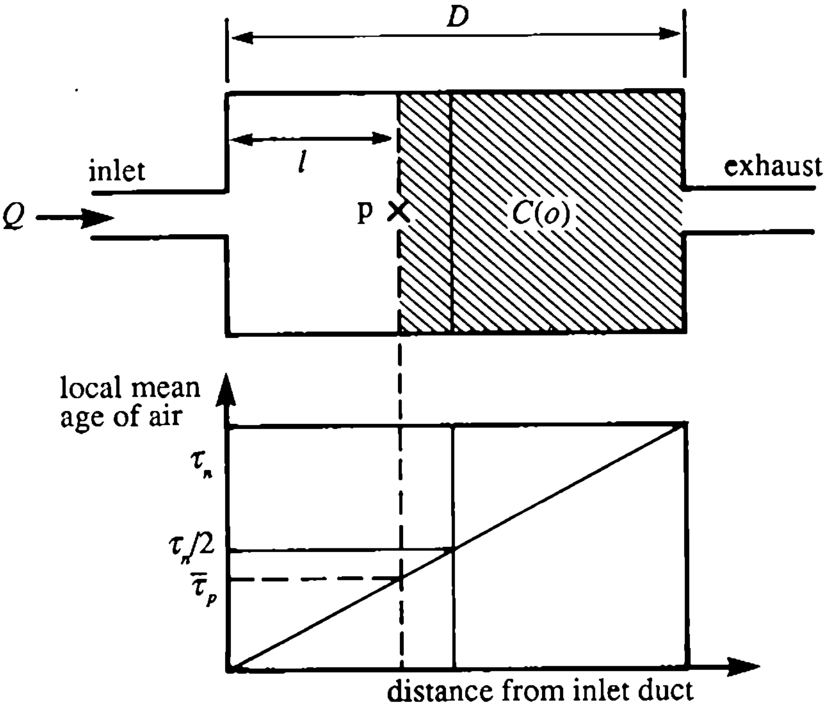

Fig. 7 Variation of local mean age with position

Setting *l* = D shows that the local mean age of air at the exhaust duct is equal to the nominal time constant $\tau_n = V/Q$ as expected. Therefore the room mean age of air is equal to $\tau_n/2$, i.e. the room mean age of air for piston flow is equal to the local mean age of air at a point equidistant between the inlet and exhaust ducts.

From equation 36, the air change time for the room is

$$\bar{\tau}_r = 2 < \bar{\tau} >= \tau_n$$

and from equation 37, the air change efficiency for the room

$$\varepsilon_a = \frac{\tau_n}{\bar{\tau}_r} = \frac{\tau_n}{\tau_n} = 1 \text{ or } 100\%$$

Therefore, by the time τ_n, 100% of the air in the room will have been replaced by the fresh inlet air. Also the coefficient of air change performance, which equals twice the air change efficiency, must be 2 or 200%.

8.2 Complete Mixing

Under conditions of complete mixing, incoming air continuously and uniformly mixes with the room air. This is often assumed but rarely achieved.

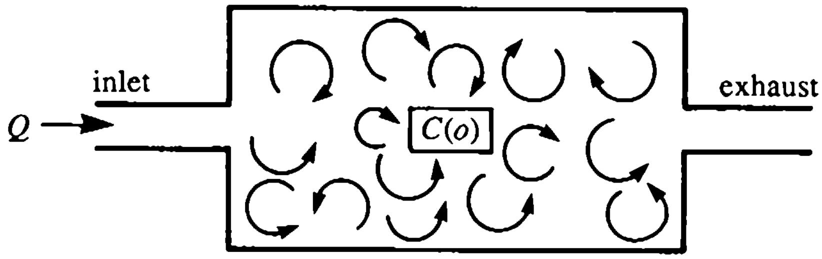

Fig. 8 Complete mixing

Assuming a tracer decay test, a plot of concentration of tracer gas against time for any point p gives

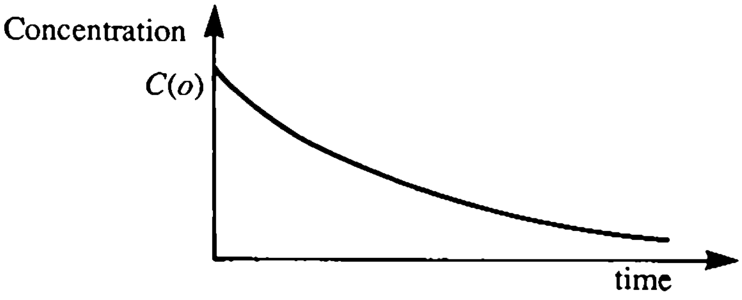

Fig. 9. Tracer decay curve for complete mixing

This is also the same for a pulse test, because as soon as the pulse is injected into the room complete mixing occurs from which a decay begins.

The tracer concentration at point p decays according to the equation

$$C_p(t) = C(o)e^{-(t/\tau_n)} \tag{41}$$

From equation 16, the local mean age of air

$$\bar{\tau}_p = \int_0^\infty \frac{C_p(t)}{C(o)}\,dt$$

Substituting using equation 41, gives

$$\bar{\tau}_p = \int_0^\infty \frac{C(o)e^{-(t/\tau_n)}}{C(o)}\,dt$$

$$= \left[-\tau_n e^{-(t/\tau_n)}\right]_0^\infty$$

$$\bar{\tau}_p = \tau_n$$

The tracer decay will be the same for all points in the room, since there is complete mixing, and therefore the local mean age of air for all points in the room will equal $\bar{\tau}_p$. From this it follows that the room mean age of air will also equal $\bar{\tau}_p$.

Therefore the room mean age of air

$$< \bar{\tau} > = \tau_n$$

This result can also be calculated from equation 34 where the room mean age of air for a tracer decay test is given by

$$< \bar{\tau} > = \frac{Q}{V}\int_0^\infty t\,\frac{C_e(t)}{C(o)}\,dt$$

Therefore substituting using equation 41

$$< \bar{\tau} > = \frac{Q}{V}\int_0^\infty \frac{t.C(o)e^{-(t/\tau_n)}}{C(o)}\,dt$$

Integrating by parts gives

$$< \bar{\tau} > = \frac{Q}{V}\,\tau_n\left[-\tau_n . e^{-(t/\tau_n)}\right]_0^{\infty}$$

Therefore

$$< \bar{\tau} > = \tau_n$$

The same result can also be achieved using the tracer step-up method where

$$< \bar{\tau} > = \frac{Q}{V}\int_0^{\infty} t\left(1 - \frac{C_e(t)}{C_s}\right)dt$$

for a tracer step-up test, tracer concentration at the exhaust

$$C_e(t) = C_s - C_s e^{-(t/\tau_n)}$$

Therefore

$$< \bar{\tau} > = \frac{Q}{V}\int_0^{\infty}\left(1 - \left(\frac{C_s - C_s e^{-(t/\tau_n)}}{C_s}\right)\right)dt$$

$$= \frac{Q}{V}\int_0^{\infty} t.e^{-(t/\tau_n)}dt$$

Integrating gives

$$< \bar{\tau} > = \tau_n$$

The air change time for the room, for complete mixing can then be calculated using equation 38, where

$$\bar{\tau}_r = 2 < \bar{\tau} > = 2\tau_n$$

Therefore, from equation 39

$$\varepsilon_a = \frac{\tau_n}{\bar{\tau}_r} = \frac{\tau_n}{2\tau_n} = 0 \cdot 5 \text{ or } 50\%$$

Therefore, under conditions of complete mixing the air change efficiency is 50%, and the coefficient of air change performance, which equals twice the air change efficiency, must be 1 or 100%.

9. METHODS OF MEASUREMENT

The three main methods of measurement of the room mean age and the local mean age of air, follow directly from the previously described theory. These are the pulse method, the tracer step-up method and the tracer decay method.

9.1 The Pulse Method

A small amount of tracer gas is briefly injected into the supply duct of the room and the resulting rises and decays of tracer concentration are continuously measured at points of interest. It is, however, important to ensure that the tracer gas and intake air are homogeneously mixed before leaving the inlet duct and entering the room.

Tracer gas concentrations recorded at the exhaust duct will give the room mean age of air when equation 29 is applied, and from this and from the nominal time constant for the room the air change efficiency can be calculated.

9.2 Tracer Step-Up Method

Tracer gas is injected into the supply duct at a constant rate and the resulting increases in tracer concentrations at points within the room are measured. Once again it is essential that the tracer gas and inlet air are uniformly mixed before entering the room. The start of admission of tracer gas into the room is designated as time $t = 0$ and measurements should be taken until equilibrium conditions of tracer concentration within the room are approached since this concentration will not then change further over time.

Measurements taken at the exhaust duct will give the room mean age of air when equation 31 is applied and from this the air change efficiency for the room can be calculated.

9.3 The Tracer Decay Method

This is the most widely used method of measuring the room mean age of air and thus determining the air change efficiency of the room.

Tracer gas is injected into the room and then mixed to a uniform concentration $C(o)$. The mixing fans are then switched off and the tracer gas is allowed to decay. Tracer concentrations are continuously recorded at several points within the room, for local age determination, and at the exhaust duct for the room mean age determination using equation 34 which then can be used to calculate the air change efficiency for the room.

Nordtest reports, NTVVS019: Ventilation air – local mean age and NTVVS047: Ventilating air – mean age of air, illustrate tracer decay tests to measure the local and room mean age of air and thus calculate the air exchange efficiency and local air exchange index.

10. RANGE OF APPLICATION

10.1 The Use of Air Change Efficiency Terms

The local mean ages at specific points can be compared to give a measure of the spatial variations of air distribution within a room. Points close to the inlet duct will have short local mean ages and points near the exhaust duct will generally have longer local mean ages. Local mean ages apply only to the room in which they have been measured and local mean ages for points in different rooms are not comparable.

The local air change index is the ratio of the local mean age that would exist if the air in the room were completely mixed (i.e. τ_n) to the local mean age which is actually measured at a point (i.e. $\bar{\tau}_p$). This index also gives a measure of spatial variations of air distribution in a room. However, because it is a ratio with τ_n, values obtained in different rooms can be compared.

The room mean age is the average value of the local mean ages at all points within a room. By comparing local mean ages with the room mean age, points where the supply of fresh air is above or below average can be identified. Room mean ages for different rooms cannot be compared directly.

The air change efficiency is the ratio of the room mean age that would exist if the air in the room were completely mixed (i.e. τ_n) to the air change time of the room ($\bar{\tau}_r$). Hence values obtained in different rooms are comparable. Furthermore the value of 50% indicates the fully mixed condition and the value of 100% implies piston flow. These values, when compared with the measured value for a particular room, provide an indication of the nature of air distribution in that room.

10.2 Single Inlet/Single Exhaust Case

Measurement of the ages of air, local and room mean, for the single inlet/single exhaust case, arise directly from the theory previously discussed.

Any of the three methods of measurement, pulse, tracer step-up or tracer decay, may be used to find the tracer concentrations at the exhaust duct and/or at specific points within the room. Then, by use of the correct equations depending upon which measurement method was used, the room mean age and local mean age of air can be found from the tracer concentrations.

The room mean age can then be used to find the air change efficiency of the room and the local mean age leads to an air change index value for the particular point being measured.

10.3 Single Inlet/Multiple Exhaust Case

Once again any of the three tracer gas methods may be used to find the room mean age and local mean ages of air.

However, to find the room mean age, tracer gas concentration measurements and the volumetric flow rates of air in each exhaust duct must be known. The concentrations are then volume weighted in the calculation of the room mean age of air. An example of tracer gas measurement in several exhaust ducts is given in the Nordtest report NTVVS047 [3], where the room mean age is calculated using the following equation

$$<\bar{\tau}>=\frac{q_1\mu_1^1+q_2\mu_2^1+\ldots\ldots+q_n\mu_n^1}{q'\mu_1^0+q_2\mu_2^0+\ldots\ldots+q_n\mu_n^0}$$

where q_d = the volumetric flow rate (m^3/h) in duct number d

n = the total number of extract ducts

μ_d^0 = the 0:th moment of the tracer concentration versus time graph (i.e. the area under the curve)

μ_d^1 = the 1st moment of the tracer concentration versus time graph.

Calculations for the local mean age of air at a particular point remain unchanged as for case 10.1 single inlet/single exhaust case.

10.4 Multiple Inlet/Single Exhaust

The tracer decay method should be used to determine the room mean age and local mean age of the air in a room with multiple inlets and only a single exhaust. For the pulse and step-up methods, equal concentrations of tracer gas would have to be injected into each inlet at equal rates and in practice this would be more difficult to achieve than running a simple tracer decay test.

Again tracer concentrations measured at the exhaust can be used to find the room mean age of air, and the concentrations measured at specific points within the room lead to the local mean age values at those points under consideration.

10.5 Multiple Inlet/Multiple Exhaust

For this case, the tracer decay method should again be used. The determination of the local mean ages of air at specific points within the room remains the same as for case 8.3, but for the calculation of the room mean age, tracer concentrations and volumetric flow rates should be recorded in each exhaust duct. The tracer concentrations are then volume weighted for the calculation of the room mean age as for case 8.2 single inlet/multiple exhaust.

10.6 Infiltration/Exfiltration

The theory and applications previously described assume that all the ventilation airflow which enters or leaves a room does so via the mechanical ventilation system; no allowances are made for infiltration/exfiltration.

The local mean age of air can still be determined from a tracer decay test, but the pulse method and tracer step-up methods will not be so accurate.

The room mean age cannot accurately be found where infiltration/exfiltration is a problem since tracer concentration measurements cannot be recorded at all points of exhaust from the room.

Freeman *et al* [5] have studied air change efficiency in naturally ventilated dwellings.

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APPENDIX A

The integral $\int_0^\infty x^{k+1} f'(x)\,dx = I$

Where $f(x)$ tends to A as x approaches infinity
$f(x)$ tends to B as x approaches zero
and $f(x)$ is continuous in the interval zero to infinity.

I can be integrated by parts between the limits of 0 and N, where N is a finite, arbitrarily large number. Therefore,

$$I = \left[x^{k+1} f(x)\right]_0^N - \int_0^N (k+1) x^k f(x)\,dx$$

$$= N^{k+1} f(N) - \int_0^N (k+1) x^k f(x)\,dx$$

Note that

$$\int_0^N (k+1) x^k f(N)\,dx = \left[x^{k+1} f(N)\right]_0^N = N^{k+1} f(N)$$

Hence

$$I = \int_0^N (k+1) x^k f(N)\,dx - \int_0^N (k+1) x^k f(x)\,dx$$

$$= (k+1) \int_0^N x^k \left(f(N) - f(x)\right) dx$$

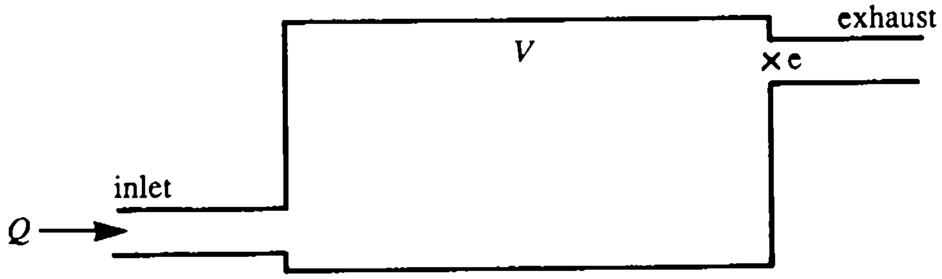
As $N \longrightarrow \infty, f(N) \longrightarrow A$. Thus

$$I = \int_0^\infty x^{k+1} f'(x)\,dx = (k+1) \int_0^\infty x^k \left(A - f(x)\right) dx$$

APPENDIX B

Air Change Time ($\bar{\tau}_r$)

Consider a tracer step-up last, where tracer gas of concentration C_s is continuously injected into a room at a rate of $\dot{m}$ (m^3/s)



The concentration of tracer C_s, in the supply duct

$$C_s = \frac{\dot{m}}{Q} \tag{B.1}$$

and the concentration of tracer in the exhaust duct at time t = ∞

$$C_e(\infty) = C_s \tag{B.2}$$

Constructing a mass balance equation for the system, the amount of tracer present in the room

$$M(t) = \dot{m}t - Q\int_0^t C_e(t)dt \tag{B.3}$$

However, from equation 7

$$C_e(t) = C_s\int_0^t A_e(t)dt = C_sF_e(t) \tag{B.4}$$

Therefore

$$M(t) = \dot{m}t - QC_s\int_0^t F_e(t)dt \tag{B.5}$$

Rearranging this and substituting using equation (B.1) gives

$$t-\int_0^t F_s(t)dt=\frac{M(t)}{QC_s} \tag{B.6}$$

At time t = ∞, the amount of tracer in the system

$$M(\infty)=VC_s \tag{B.7}$$

Therefore

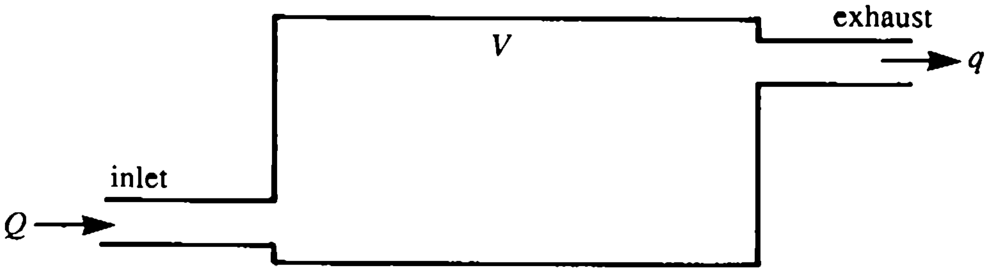
$$t-\int_0^t F_s(t)dt=\frac{V}{Q}\frac{M(t)}{M(\infty)}=\frac{V}{Q}\int_0^t \phi(t)dt \tag{B.8}$$

where $\frac{M(t)}{M(\infty)}$ is the total material accumulated up to time t as a fraction of the final steady value, and is therefore the cumulative distribution function, and $\phi(t)$ is the corresponding frequency distribution.

Differentiating gives

$$1-F_s(t)=\frac{V}{Q}\phi(t)\,dt \tag{B.9}$$

Then consider the residual time distribution of contaminant in the room



Let $f(t)$ be the exhaust age distribution, $\dot{m}$ be the production rate of contaminant in the inlet duct and q be the rate of outflow of contaminant at the exhaust duct.

Then the rate of outflow of contaminant occurring between the times t and (t + δt), is

$$q(t) = \dot{m} f(t) dt \tag{B.10}$$

The quantity of material in the room associated with this exhaust time of t, must be

$$t.q(t) = \dot{m}.t f(t) dt \tag{B.11}$$

This is the fraction of the total material in the room which has this particular exhaust age (i.e. it will be resident in the room for this length of time)

Therefore,

$$dM(t) = t.q(t) = \dot{m}.t.f(t) dt \tag{B.12}$$

The fraction of the total final room content with an exhaust time between t and (t + δt), is χ(t)dt, and is related to *dM*(*t*) by

$$\chi(t) dt = \frac{dM(t)}{M(\infty)} = \frac{\dot{m} t f(t) dt}{M(\infty)} \tag{B.13}$$

But $C_s = \frac{\dot{m}}{Q}$ and $M(\infty) = VC_s$ therefore

$$\chi(t) dt = \frac{Q}{V} t f(t) dt \tag{B.14}$$

The average exhaust time $\overline{\tau}_r$ is therefore given by

$$\overline{\tau}_r = \int_0^\infty t\chi(t) dt = \frac{Q}{V}\int_0^\infty t^2 f(t) dt = \frac{2Q}{V}\int_0^\infty t(1 - F(t)) dt$$

where $\overline{\tau}_r$ can also be called the air change time since the time it takes to exhaust all the air in a room is equal to the time it takes to change the air in the room.

Then from equation B.9

$$\bar{\tau}_r = 2\int_0^{\infty} t\phi(t)\,dt$$

By definition, the first moment of the frequency distribution is the room mean age,

$$< \bar{\tau} > = \int_0^{\infty} t\phi(t)\,dt$$

Therefore the air change time equals twice the mean age of the air in the room

$$\bar{\tau}_r = 2 < \bar{\tau} >$$

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